

Mathematics Design Collaborative  
State of Georgia Department of Education



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# Graphing Logarithmic and Exponential Functions

## *Graphing Logarithmic and Exponential Functions*

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# INTRODUCTION TO THIS FORMATIVE ASSESSMENT LESSON

## MATHEMATICAL GOALS

This lesson unit is intended to help you assess how well students are able to:

- ✎ Graph exponential and logarithmic functions
- ✎ Determine x and y intercepts and asymptotes of exponential and logarithmic functions
- ✎ Describe the domain, range and end behaviors of exponential and logarithmic functions
- ✎ Understand that exponential and logarithmic functions are inverses of each other

## GEORGIA STANDARDS OF EXCELLENCE

This lesson involves mathematical content in the standards from across the grades, with emphasis on::

- ✎ **MGSE9-12.F.IF.7e** Graph exponential and logarithmic functions, showing intercepts and end behavior, ~~and trigonometric functions, showing period, midline, and amplitude.~~
- ✎ **MGSE9-12.F.BF.5** Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.
- ✎ **SMP1**-Make sense of problems and persevere in solving them.
- ✎ **SMP2-2.** Reason abstractly and quantitatively.
- ✎ **SMP3**- Construct viable arguments and critique the reasoning of others.
- ✎ **SMP6** - Attend to precision

## INTRODUCTION

This lesson is structured in the following way:

### Before the Lesson,

Students work individually on an assessment task that is designed to reveal their current understandings and difficulties. You then review their work, and create questions for students to answer in order to improve their solutions.

### At the Start of the Lesson,

Students work in pairs on a collaborative discussion task in which they match exponential and logarithmic functions to their graphs, describe their characteristics, and identify inverse functions. Upon completing their assignment, students work in their same groups to analyze the work of other groups within the class.

### During the Lesson,

Students explain their thought processes and the strategies they used in completing their task.

### After the Whole-Group Class Discussion,

Students return to their original assessment tasks, and try to improve their own responses.

## MATERIALS REQUIRED

Each individual student will need:

- ✎ White board, dry erase marker & eraser
- ✎ Glue
- ✎ Chart paper, poster, or large construction paper
- ✎ A copy of Set 1 – Exponential Functions (cut apart)

- ✂ A copy of Set 2 – Logarithmic Functions (cut apart)

## TEACHER PREP REQUIRED

Teacher, be advised that prior to the lesson, the following preparations/copies will need to be made:

- ✂ A copy of Set 1 – Exponential Functions (cut apart)
- ✂ A copy of Set 2 – Logarithmic Functions (cut apart)

## TIME NEEDED:

For Pre-Assessment: 15 min

For Lesson: 75-80 min

For Post: 15 min

## FRAMING FOR THE TEACHER:

This Formative Assessment Lesson is to be used two-thirds of the way through the Logarithm/Exponential Unit, when students understand how to graph both and how to identify key features of these graphs. Oftentimes students are able to slice and dice the intricacies of these two, but they are not clear about the relationship between the two. This lesson was carefully crafted so that the inverses of the exponential functions shown are the logarithmic functions. Students should be able to discuss this relationship with the teacher at the conclusion of the lesson.

## FRAMING FOR THE STUDENTS:

Say to the students:

*This activity will take about 2-3 days for us to complete.*

*The reason we are doing this is to be sure that you understand: graphing exponentials and logs before we move on to a new idea.*

*You will have a chance to work with a partner to correct any misconceptions that you may have. After the partner work, you will be able to show me what you have learned!*

## PRE-ASSESSMENT BEFORE THE LESSON

### ASSESSMENT TASK: Graphing Logarithmic/Exponential Functions

Time This Should Take: 15 minutes

Have the students do this task in class or for homework, a day or more before the formative assessment lesson. This will give you an opportunity to assess the work, and to find out the kinds of difficulties students have with it. You will then be able to target your help more effectively in the follow-up lesson.

Give each student a copy of Graphing Exponential and Logarithmic Functions:

Briefly introduce the task and help the class to understand the problem and its context.

*Spend 15 minutes working individually on this task. Read through the task and try to answer it as carefully as you can. Show all your work so that I can understand your reasoning. Don't worry if you can't complete everything. There will be a lesson that should help you understand these concepts better. Your goal is to be able to confidently answer questions similar to these by the end of the next lesson.*

Students should do their best to answer these questions, without teacher assistance. It is important that students are allowed to answer the questions on their own so that the results show what students truly do not understand.

Students should not worry too much if they cannot understand or do everything on the pre-assessment, because in the next lesson they will engage in a task which is designed to help them.. Explain to students that by the end of the next lesson, they should expect to be able to answer questions such as these confidently.


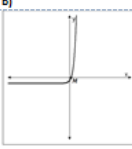

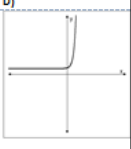
This is their goal.

G11.U4 or Accelerated G10.U3

Pre-Post Assessment: Graphing Exponential & Logarithmic Functions

Student Name: \_\_\_\_\_

**1) Match each graph below to its function.**

A)	B)	C)	D)
			

a)  $f(x) = 2^{x+4} - 4$  matches graph \_\_\_\_\_. I can tell because \_\_\_\_\_.

b)  $f(x) = 2^{x+4} + 4$  matches graph \_\_\_\_\_. I can tell because \_\_\_\_\_.

c)  $f(x) = \log_2(x+1) - 4$  matches graph \_\_\_\_\_. I can tell because \_\_\_\_\_.

d)  $f(x) = \log_2(x-1) + 4$  matches graph \_\_\_\_\_. I can tell because \_\_\_\_\_.

2) What is the ordered pair for point M on the graph above? \_\_\_\_\_. Explain how you found your answer. \_\_\_\_\_.

3) a) The asymptote of  $g(x) = 4^{x+8}$  is  $y = -8$ . Explain algebraically why this is the case. \_\_\_\_\_.

b) How can the asymptote of  $g^{-1}(x)$  be found? \_\_\_\_\_.

4) Ms. Math asked Becky to find the Domain and Range of  $g(x)$ , and she asked Sherry to find the Domain and Range of  $g^{-1}(x)$ . Together they figured out a way to cut their work in half. What did they do? \_\_\_\_\_.











## **COLLABORATION TIME/READING STUDENTS RESPONSES**

### **You Will Not “Grade” These!**

Collect students’ responses to the task. It is helpful to read students’ responses with colleagues who are also analyzing student work. Make notes (on your own paper, not on their pre-assessment) about what their work reveals about their current levels of understanding, and their approaches to the task. You will find that the misconceptions reveal themselves and often take similar paths from one student to another, and even from one teacher to another. Some misconceptions seem to arise very organically in students’ thinking. Pair students in the same classes with other students who have similar misconceptions. This will help you to address the issues in fewer steps, since they’ll be together. (Note: pairs are better than larger groups for FAL’s because both must participate in order to discuss!)

You will begin to construct Socrates-style questions to try and elicit understanding from students. We suggest you write a list of your own questions; however some guiding questions and prompts are also listed below as a jumping-off point.

### **GUIDING QUESTIONS**

<b><i>COMMON ISSUES</i></b>	<b><i>SUGGESTED QUESTIONS AND PROMPTS</i></b>
Student has a hard time getting started	 <i>What would happen if you just plugged numbers in place of <math>x</math>?</i>  <i>What would parent function look like without plus four, minus one, etc...?</i>
Students match logarithmic function to exponential graph, and vice-versa.	 <i>What does the graph of an exponential function look like?</i>  <i>What does the graph of a logarithmic function look like?</i>
Students match exponential graph to incorrect exponential function.	 <i>How can points on the graph be found?</i>
Students cannot label ordered pairs on graphs.	 <i>How are <math>x</math> and <math>y</math> intercepts found?</i>
Students cannot write a function that doesn’t have an $x$ -intercept.	 <i>What changes in the function cause the graph to move?</i>
Students cannot write a pair of inverse functions.	 <i>What are the relationships between inverse functions?</i>
Student has a hard time getting started	 <i>What would happen if you just plugged numbers in the place of <math>x</math>?</i>  <i>What would the parent function look like without the plus four, minus one, etc...?</i>

# LESSON DAY

## SUGGESTED LESSON OUTLINE:

### Part 1: Whole-Class Introduction:

**Time to Allot: (10 minutes)**

Display the “Warm Up” question provided.

Give each student a mini-whiteboard, pen and eraser.

Introduce the lesson with:

*Today, we are going to look at the key features of exponential & logarithmic curves and the relationships between exponential & logarithmic functions. On your mini-whiteboards, write an exponential growth function  $f(x)$  and sketch its graph.*

Allow students to work for a few minutes and then ask them to show you their whiteboards. Be selective as to which student you ask to explain his or her graphs. Look for several different graphs. Specifically graphs with:

- A positive asymptote
- A negative asymptote
- Different bases

*What are the common features of these graphs? What part of the function creates these common features?*

*What is different about the graphs? What part(s) of the functions cause these differences?*

Elicit responses from the class and try to keep your own interventions to a minimum. Encourage students to use mathematical terms such as roots, y-intercepts, asymptote, domain, range, and end behaviors. Make sure that students discuss the movement of the asymptotes, and the rates of change of the different graphs and how the functions affect these features.

*Now on your mini-whiteboards, sketch the graph of your functions' inverse.*

*Then write the function  $f^{-1}(x)$ .*

Allow students to work for a few minutes and then ask them to show you their whiteboards.

Ask several students to explain the relationship between their graphs of  $f(x)$  and  $f^{-1}(x)$ . Encourage students to address the relationships between the domains and ranges, x and y intercepts, and the asymptotes. Also have students discuss the relationship between the functions  $f(x)$  and  $f^{-1}(x)$ .

*Now on your mini-whiteboards, write an exponential decay function  $g(x)$  and sketch its graph.*

Allow students to work for a few minutes and then ask them to show you their whiteboards.

Be selective as to which student you ask to explain his or her graphs. Look for several different graphs. Specifically graphs with:

- A positive asymptote
- A negative asymptote
- Different bases

*What are the common features of these graphs? What part of the function creates these common features?*

*What is different about the graphs? What part(s) of the functions cause these differences?*

Elicit responses from the class and try to keep your own interventions to a minimum. Encourage students to use mathematical terms such as roots, y-intercepts, asymptote, domain, range, and end behaviors. Make sure that students discuss the movement of the asymptotes, and the rates of change of the different graphs and how the functions affect these features.

*Now on your mini-whiteboards, sketch the graph of your functions' inverse.*

*Then write the function  $g^{-1}(x)$ .*

Allow students to work for a few minutes and then ask them to show you their whiteboards.

Ask several students to explain the relationship between their graphs of  $g(x)$  and  $g^{-1}(x)$ . Encourage students to address the relationships between the domains and ranges, x and y intercepts, and the asymptotes. Also have students discuss the relationship between the functions  $g(x)$  and  $g^{-1}(x)$ .

**Part 2: Collaborative Activity:****Time to Allot: ( 60 minutes)**

Put students into their pairs according to your analysis of student errors.

Do/Say the Following:

**Group Work (30 minutes)**

Have students work in groups of 2 or 3 to complete the activity. It is recommended that the teacher group the students homogeneously based on misconceptions found in the pre-assessment.

Give each group the card set, along with tape or glue to hold the graphs and functions together as they are matched and a sheet of poster/chart paper. (Note: Two versions of the cards are included to allow for differentiation within the classroom.)

*You are now going to work in your groups to match each function with its graph. Once the graphs and functions are matched, tape or glue the pairs together then identify missing characteristics on the function card.*

*Now match each function/graph pair to its inverse function/graph. Then tape/glue the inverse pairs to your poster, leaving space beside each pair. In a sentence or two, beside each pair of functions explain how you determined that they are inverses*

The purpose of this structured group work is to encourage students to engage with each other's explanations and take responsibility for each other understands.

You have two tasks during the small-group work: to make a note of student approaches to the task, and to support student reasoning. Listen and watch students carefully. Note different student approaches to the task and any common mistakes. Also notice the ways students check to see if their match is correct and how they explain and justify a match to each other. You can use this information to focus a plenary whole-class discussion. In addition, try not to make suggestions that move students towards a particular match. Instead, ask questions to help students to reason together. If you find one student has produced a solution for a particular match, challenge another student in the group to provide an explanation.

**Ex. John matched these cards. Sharon, why do you think John matched these two cards?**

If you find students have difficulty articulating their decisions, use the sheet Suggested questions and prompts to support your own questioning of students. If the whole class is struggling on the same issue, you could write a couple of questions on the board and hold an interim, whole-class discussion. You could also ask students who performed well in the assessment to help struggling students.

**Sharing posters (5 minutes)**

As students finish their posters, ask one student from each group to visit another group's poster. (The posters should be the same as they are constructed from the same data set.)

Give the students these instructions:

*If you are staying at your desk, be ready to explain the reasons for your group's matches.*

*If you are visiting another group, write your card placements on a piece of paper. Go to another group's desk and check to see which matches are different from your own.*

*If there are differences, ask for an explanation. If you still don't agree, explain your own thinking. When you return to your own desk, you need to consider as a group whether to make any changes to your own poster.*

Students may now want to make changes to their poster.



**Whole-class discussion: Exponential & Logarithmic Functions (20 - 30 minutes)**

Once students have finished sharing posters, organize a whole-class discussion.

- 👤 Invite pairs of students to describe one pair of cards (exponential function & graph) that they think they have matched correctly, and the reasoning they employed. Also, have them explain how they determined the various characteristics related to their function and graph.

Encourage other students to challenge their explanations.

- 👤 Once students have discussed their results for the exponential functions, complete the same process for the logarithmic functions. Make sure to discuss the graph, as well as all characteristics.
- 👤 Finally, have students share exponential/logarithmic pairs that they think are inverse functions. Ask them to explain their reasoning both algebraically and graphically. Again, encourage other students to challenge their explanations.

To wrap up the class discussion, give each student a mini-whiteboard, a pen, and an eraser.

- 👤 Give students 2 minutes to write and graph an increasing exponential function with an asymptote at  $y = -3$ .
- 👤 After 2 minutes, have them trade white boards with their partner. The partners should silently check each others' work (allow 1-2 minutes for this), then take 3-4 minutes to discuss their functions and make corrections to their work.
- 👤 Select 3 – 4 students to share their functions and graphs. Ask the class to analyze if the function is increasing, does it have the correct asymptote, does the graph match the function?
- 👤 Give the students 4-5 minutes to find and graph the inverse of the exponential function they just wrote.
- 👤 After 4 -5 minutes, have them trade white boards with their partner. The partners should silently check each others' work (allow 1-2 minutes for this), then take 3-4 minutes to discuss their functions and make corrections to their work.
- 👤 Select 3 – 4 students to share their functions and graphs. Ask the class to analyze if the function is the correct inverse, does it have the correct asymptote, is it graphed correctly?

**Poster Sharing**

One person from each group stays with the poster, and one person moves to analyze the work of other groups. Does your work match that of the other groups'?

If you are staying at your desk, be ready to explain the reasons for your group's matches.

If you are visiting another group, write your card placements on a piece of paper. Go to another group's desk and check to see which matches are different from your own. If there are differences, ask for an explanation. If you still don't agree, explain your own thinking. When you return to your own desk, you need to consider as a group whether to make any changes to your own poster. You should visit at least 2 groups.

During the Collaborative Activity, the Teacher has 3 tasks:

- 👤 Circulate to students' whose errors you noted from the pre-assessment and support their reasoning with your guiding questions.
- 👤 Circulate to other students also to support their reason in the same way.
- 👤 Make a note of student approaches for the summary (plenary discussion). Some students have interesting and novel solutions!



**Part 3: Plenary (Summary) Discussion:****Time to Allot: ( 20-30 minutes)**

Gather students together, share solutions. Discussion prompts should be made up of your original guiding questions and notes about student approaches. Some other discussion prompts are listed below:

*NOTE: “Scribing” helps to increase student buy-in and participation. When a student answers your question, write the student’s name on the board and scribe his/her response quickly. You will find that students volunteer more often when they know you will scribe their responses – this practice will keep the discussions lively and active!*

Bring the class together and use the following to guide discussion:

Once students have finished sharing posters, organize a whole-class discussion.

- ✎ Invite pairs of students to describe one pair of cards (exponential function & graph) that they think they have matched correctly, and the reasoning they employed. Also, have them explain how they determined the various characteristics related to their function and graph.
- ✎ Encourage other students to challenge their explanations.
- ✎ Once students have discussed their results for the exponential functions, complete the same process for the logarithmic functions. Make sure to discuss the graph, as well as all characteristics.
- ✎ Finally, have students share exponential/logarithmic pairs that they think are inverse functions. Ask them to explain their reasoning both algebraically and graphically. Again, encourage other students to challenge their explanations.

To wrap up the class discussion, give each student a mini-whiteboard, a pen, and an eraser.

- ✎ Give students 2 minutes to write and graph an increasing exponential function with an asymptote at  $y = -3$ .
- ✎ After 2 minutes, have them trade white boards with their partner. The partners should silently check each others’ work (allow 1-2 minutes for this), then take 3-4 minutes to discuss their functions and make corrections to their work.
- ✎ Select 3 – 4 students to share their functions and graphs. Ask the class to analyze if the function is increasing, does it have the correct asymptote, does the graph match the function?
- ✎ Give the students 4-5 minutes to find and graph the inverse of the exponential function they just wrote.
- ✎ After 4 -5 minutes, have them trade white boards with their partner. The partners should silently check each others’ work (allow 1-2 minutes for this), then take 3-4 minutes to discuss their functions and make corrections to their work.
- ✎ Select 3 – 4 students to share their functions and graphs. Ask the class to analyze if the function is the correct inverse, does it have the correct asymptote, is it graphed correctly?

**Part 4: Improving Solutions to the Assessment Task****Time to Allot: (15-20 minutes)**

The Shell MAP Centre advises handing students their original assessment tasks back to guide their responses to their new Post-Assessment (which is sometimes the exact same as the Pre-Assessment). In practice, some teachers find that students mindlessly transfer incorrect answers from their Pre- to their Post-Assessment, assuming that no “X” mark means that it must have been right. . Until students become accustomed to **UNGRADED FORMATIVE** assessments, they may naturally do this. Teachers often report success by handing students a list of the guiding questions to keep in mind while they improve their solutions.

Practice will make perfect, and teachers should do what makes them most comfortable with their students/finds and kills misconceptions!

Give students a second blank copy of their tasks back.

Return to the students their original assessment task if you choose (some teachers choose not to return the original assessment task because students tend to just copy the old answers without realizing that they had incorrect answers previously; they misinterpret a lack of “X” marks as meaning they got the problems all correct – it may be worthwhile to mention to students who are new to FAL’s that the lack of an “X” does not mean they got it right if you do choose to give students their original tasks back).

*Look at your original responses and think about what you have learned this lesson.*

*Using what you have learned, try to improve your work.*

If you have not added questions to individual pieces of work then write your list of questions on the board.

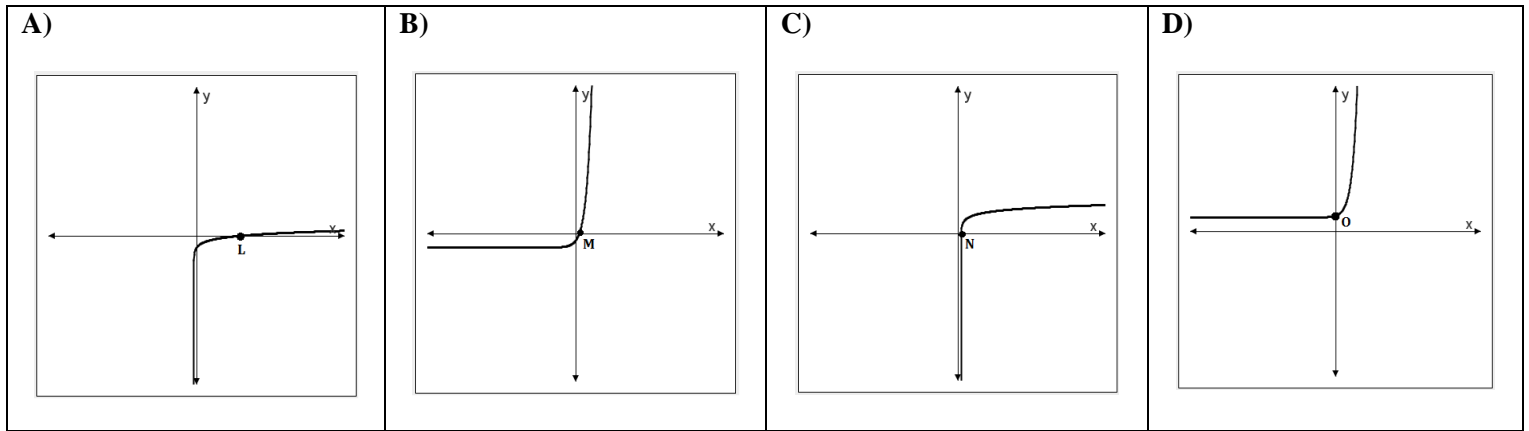
Students should select from this list only the questions they think are appropriate to their own work.

If you find you are running out of time, then you could set this task in the next lesson or for homework.

## PRE-ASSESSMENT (Answer Key)

### ASSESSMENT TASK: Graphing Exponential & Logarithmic Functions

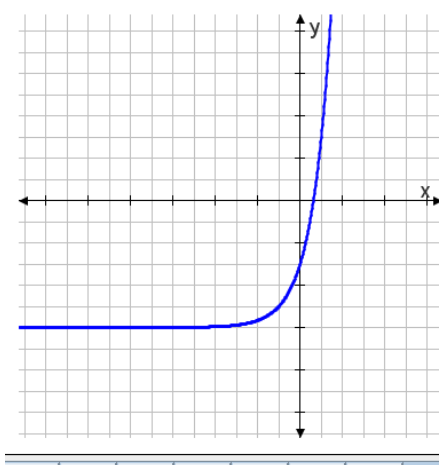
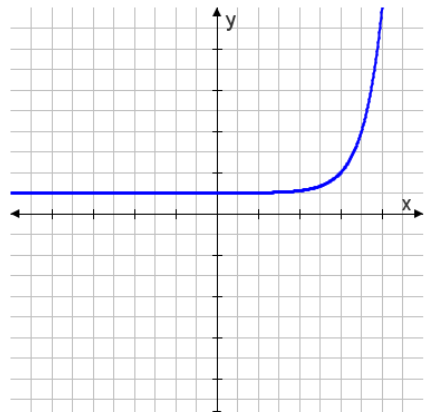
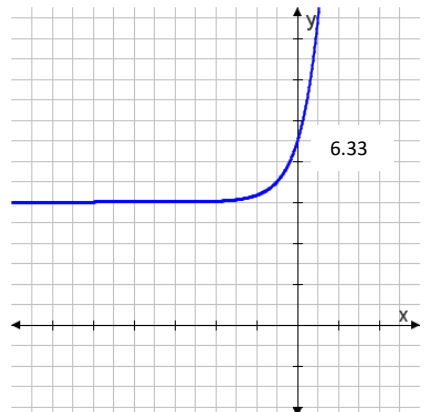
Match each graph below to its function.



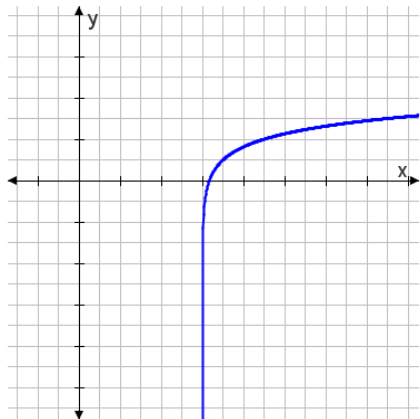
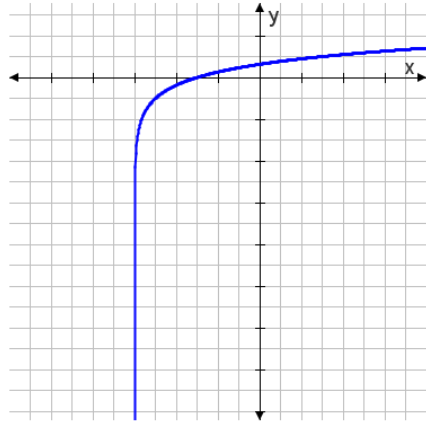
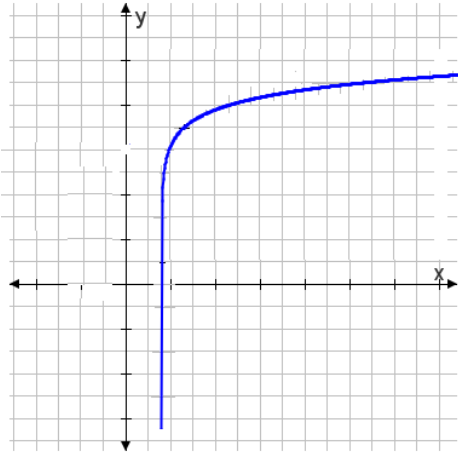
- 1)  $f(x) = 2^{x+1} - 4$  matches graph B. I can tell because exponential equation, which increases without bound (since the base > 1)...anything to the zero power is one, so normal exponential functions cross at (0,1)...but for this one to have a zero exponent, the x would have to be negative one, so that shifts left one unit. Then the minus four shifts down four units. (B) is the only one that does this.
- 2)  $f(x) = 2^{x-1} + 4$  matches graph D. I can tell because This is another exponential equation. It will increase like the one above. It is shifted right one and up four. So that's D.
- 3)  $f(x) = \log_2(x+1) - 4$  matches graph A. I can tell because This is a logarithmic equation. They intersect at (1,0), since they are inverses of exponentials. I would typically look for a graph with (1,0) (2,1) and (4,2) in it. But this one is shifted left one, down four. So it's going to be below where it normally is. (points would be (0, -4), (1, -3) and \*(3, -2)...)The one that hits at (zero, a negative number) is A.
- 4)  $f(x) = \log_2(x - 1) + 4$  matches graph C. I can tell because shifted right one and up four. x-intercept point of all non-transformed logs (1,0), and then shift it right one and up four, you'd be at (2,4).
- 5) What is the ordered pair for point M on the graph above? (1,0) Explain how you found your answer.  $f(x) = 2^{x+1} - 4$ . M is an x-intercept ( $f(x) = 0$ )  $\rightarrow 2^{x+1} - 4 = 0 \rightarrow 2^{x+1} = 4 \rightarrow 2^{x+1} = 2^2 \rightarrow x+1 = 2 \rightarrow x = 1$  .... (1, 0) is Point M
- 6) a) Asymptote of  $g(x) = 4^{x-1} - 8$  is  $y = -8$ . Explain algebraically why this is the case. Asymptote of  $4^x$  normally  $y=0$ , but shifted down 8. Algebraically, minus eight subtracts eight units from every y-value. So asymptote  $y = -8$ .  
 b) How can the asymptote of  $g^{-1}(x)$  be found? Switch the vertical and horizontal asymptotes of  $g(x)$  so that horizontal becomes vertical and vertical becomes horizontal. Those are the asymptotes of the inverse function.
- 7) Ms. Math asked Becky to find the Domain and Range of  $g(x)$ , and she asked Sherry to find the Domain and Range of  $g^{-1}(x)$ . Together they figured out a way to cut their work in half. What did they do? know domain and range for inverses are flip-flopped, don't need to do calculations for both; they could find Domain and Range of  $g(x)$ , then reverse them (domain becomes range and range becomes domain) for  $g^{-1}(x)$ .

## Collaborative Activity (Answer Key)

### Card Set Option 1&2 SOLUTIONS

<p>1-1</p> $f(x) = 3^{(x+1)} - 6$ <p>Domain: <u>All Real Numbers</u></p> <p>Range: <u><math>(-6, 0)</math></u></p> <p>Asymptote: <u><math>y = -6</math></u></p> <p>x-intercept: <u><math>(-0.63, 0)</math></u></p> <p>y-intercept: <u><math>(0, -3)</math></u></p> <p>End Behaviors: <math>\begin{matrix} \text{As } x \rightarrow \infty, y \rightarrow \infty \\ \text{As } x \rightarrow -\infty, y \rightarrow -6 \end{matrix}</math></p>	<p>1-b</p> 
<p>1-2</p> $f(x) = 3^{(x-6)} + 1$ <p>Domain: <u>All Real Numbers</u></p> <p>Range: <u><math>(1, \infty)</math></u></p> <p>Asymptote: <u><math>y = 1</math></u></p> <p>x-intercept: <u>none</u></p> <p>y-intercept: <u><math>\approx(0, 1.001)</math></u></p> <p>End Behaviors: <math>\begin{matrix} \text{As } x \rightarrow \infty, y \rightarrow \infty \\ \text{As } x \rightarrow -\infty, y \rightarrow 1 \end{matrix}</math></p>	<p>1-c</p> 
<p>1-3</p> $f(x) = 3^{(x-1)} + 6$ <p>Domain: <u>All Real Numbers</u></p> <p>Range: <u><math>(6, \infty)</math></u></p> <p>Asymptote: <u><math>y=6</math></u></p> <p>x-intercept: <u>NONE</u></p> <p>y-intercept: <u><math>\approx(0, 6.33)</math></u></p> <p>End Behaviors: <math>\begin{matrix} \text{As } x \rightarrow \infty, y \rightarrow \infty \\ \text{As } x \rightarrow -\infty, y \rightarrow 6 \end{matrix}</math></p>	<p>1-a</p> 

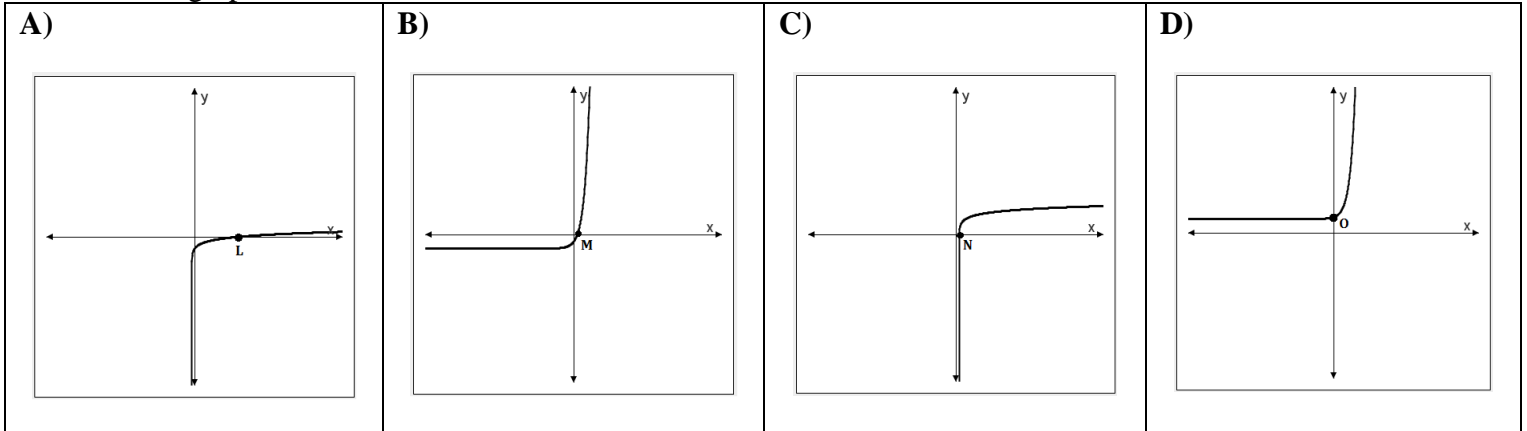
Card Set Option 1&2 **SOLUTIONS, page 2**

<p>1-4 <math>f(x) = \log_3(x - 6) + 1</math></p> <p>Domain: : <u><math>(6, \infty)</math></u></p> <p>Range: <u><math>All Real Numbers</math></u></p> <p>Asymptote: <u><math>x = 6</math></u></p> <p>x-intercept: <u><math>(6.33, 0)</math></u></p> <p>y-intercept: <u><math>NONE</math></u></p> <p>End Behaviors: <math>As x \rightarrow \infty, y \rightarrow \infty</math>  <math>As x \rightarrow -\infty, y \rightarrow -\infty</math></p>	<p>1-e</p> 
<p>1-5 <math>f(x) = \log_3(x + 6) - 1</math></p> <p>Domain: : <u><math>(-6, \infty)</math></u></p> <p>Range: <u><math>All Real Numbers</math></u></p> <p>Asymptote: <u><math>x = -6</math></u></p> <p>x-intercept: <u><math>(-3, 0)</math></u></p> <p>y-intercept: <u><math>\approx(0, 0.63)</math></u></p> <p>End Behaviors: <math>As x \rightarrow \infty, y \rightarrow \infty</math>  <math>As x \rightarrow -\infty, y \rightarrow -\infty</math></p>	<p>1-d</p> 
<p>1-6 <math>f(x) = \log_3(x - 1) + 6</math></p> <p>Domain: : <u><math>(1, \infty)</math></u></p> <p>Range <u><math>All Real Numbers</math></u></p> <p>Asymptote: <u><math>x = 1</math></u></p> <p>x-intercept: <u><math>\approx(1.001, 0)</math></u></p> <p>y-intercept: <u><math>NONE</math></u></p> <p>End Behaviors: <math>As x \rightarrow \infty, y \rightarrow \infty</math>  <math>As x \rightarrow -\infty, y \rightarrow -\infty</math></p>	<p>1-f</p> 

# PRE-ASSESSMENT

## Graphing Exponential & Logarithmic Functions

Match each graph below to its function.



1)  $f(x) = 2^{x+1} - 4$  matches graph \_\_\_\_\_. I can tell because \_\_\_\_\_

2)  $f(x) = 2^{x-1} + 4$  matches graph \_\_\_\_\_. I can tell because \_\_\_\_\_

3)  $f(x) = \log_2(x+1) - 4$  matches graph \_\_\_\_\_. I can tell because \_\_\_\_\_

4)  $f(x) = \log_2(x-1) + 4$  matches graph \_\_\_\_\_. I can tell because \_\_\_\_\_

5) What is the ordered pair for point M on the graph above? \_\_\_\_\_ Explain how you found your answer. \_\_\_\_\_

6) a) The asymptote of  $g(x) = 4^{x-1} - 8$  is  $y = -8$ . Explain *algebraically* why this is the case.

b) How can the asymptote of  $g^{-1}(x)$  be found?

7) Ms. Math asked Becky to find the Domain and Range of  $g(x)$ , and she asked Sherry to find the Domain and Range of  $g^{-1}(x)$ . Together they figured out a way to cut their work in half. What did they do?

## COLLABORATIVE ACTIVITY

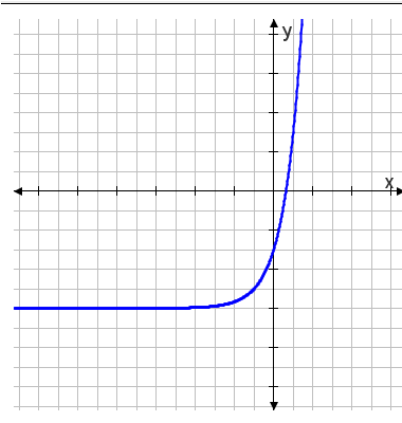
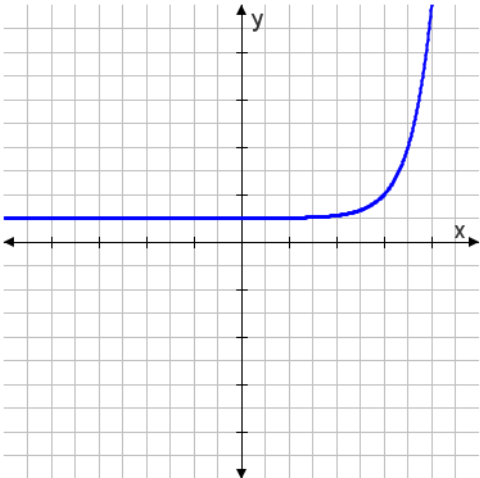
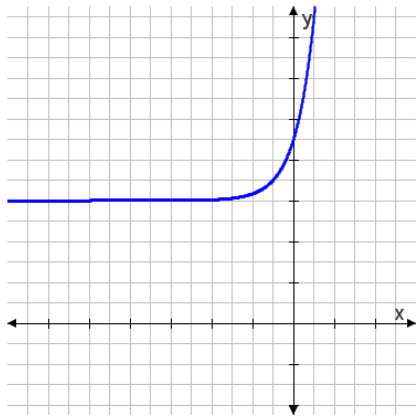
## Card Set Option 1

<p><b>1-1</b>      <math>f(x) = 3^{(x+1)} - 6</math></p> <p>Domain: _____ All Real Numbers</p> <p>Range: _____</p> <p>Asymptote: _____</p> <p>x-intercept: _____ (-0.63, 0)</p> <p>y-intercept: _____</p> <p>End Behaviors: <math>\begin{matrix} \text{As } x \rightarrow \infty, y \rightarrow \\ \text{As } x \rightarrow \infty, y \rightarrow \end{matrix}</math></p>	<p><b>1-b</b></p>
<p><b>1-2</b>      <math>f(x) = 3^{(x-6)} + 1</math></p> <p>Domain: _____</p> <p>Range: _____ [1, ∞)</p> <p>Asymptote: _____</p> <p>x-intercept: _____</p> <p>y-intercept: _____ ≈(0, 1.001)</p> <p>End Behaviors: <math>\begin{matrix} \text{As } x \rightarrow \infty, y \rightarrow \\ \text{As } x \rightarrow \infty, y \rightarrow \end{matrix}</math></p>	<p><b>1-c</b></p>
<p><b>1-3</b>      <math>f(x) = 3^{(x-1)} + 6</math></p> <p>Domain: _____</p> <p>Range: _____</p> <p>Asymptote: _____ y=6</p> <p>x-intercept: _____ NONE</p> <p>y-intercept: _____</p> <p>End Behaviors: <math>\begin{matrix} \text{As } x \rightarrow \infty, y \rightarrow \\ \text{As } x \rightarrow \infty, y \rightarrow \end{matrix}</math></p>	<p><b>1-a</b></p>

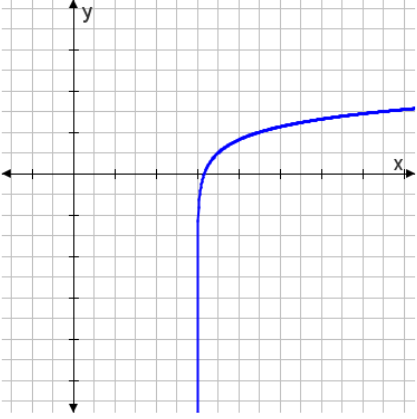
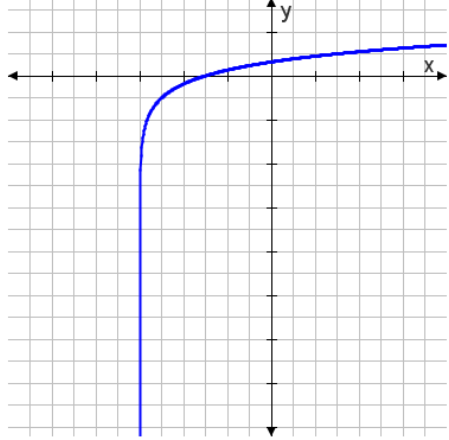
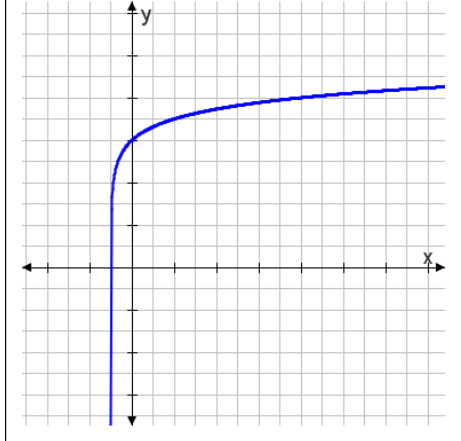




## Card Set Option 2

<p>2-1 <math>f(x) = 3^{(x+1)} - 6</math></p> <p>Domain: _____</p> <p>Range: _____</p> <p>Asymptote: _____</p> <p>x-intercept: _____</p> <p>y-intercept: _____</p> <p>End Behaviors: <math>\begin{matrix} \text{As } x \rightarrow \infty, y \rightarrow \\ \text{As } x \rightarrow \infty, y \rightarrow \end{matrix}</math></p>	<p>2-b</p> 
<p>2-2 <math>f(x) = 3^{(x-6)} + 1</math></p> <p>Domain: _____</p> <p>Range: _____</p> <p>Asymptote: _____</p> <p>x-intercept: _____</p> <p>y-intercept: _____</p> <p>End Behaviors: <math>\begin{matrix} \text{As } x \rightarrow \infty, y \rightarrow \\ \text{As } x \rightarrow \infty, y \rightarrow \end{matrix}</math></p>	<p>2-c</p> 
<p>2-3 <math>f(x) = 3^{(x-1)} + 6</math></p> <p>Domain: _____</p> <p>Range: _____</p> <p>Asymptote: _____</p> <p>x-intercept: _____</p> <p>y-intercept: _____</p> <p>End Behaviors: <math>\begin{matrix} \text{As } x \rightarrow \infty, y \rightarrow \\ \text{As } x \rightarrow \infty, y \rightarrow \end{matrix}</math></p>	<p>2-a</p> 

## Card Set Option 2, page 2

<p>2-4 <math>f(x) = \log_3(x - 6) + 1</math></p> <p>Domain: _____</p> <p>Range: _____</p> <p>Asymptote: _____</p> <p>x-intercept: _____</p> <p>y-intercept: _____</p> <p>End Behaviors: <math>\begin{matrix} \text{As } x \rightarrow \infty, y \rightarrow \\ \text{As } x \rightarrow \infty, y \rightarrow \end{matrix}</math></p>	<p>2-e</p> 
<p>2-5 <math>f(x) = \log_3(x + 6) - 1</math></p> <p>Domain: _____</p> <p>Range: _____</p> <p>Asymptote: _____</p> <p>x-intercept: _____</p> <p>y-intercept: _____</p> <p>End Behaviors: <math>\begin{matrix} \text{As } x \rightarrow \infty, y \rightarrow \\ \text{As } x \rightarrow \infty, y \rightarrow \end{matrix}</math></p>	<p>2-d</p> 
<p>2-6 <math>f(x) = \log_3(x + 1) + 6</math></p> <p>Domain: _____</p> <p>Range: _____</p> <p>Asymptote: _____</p> <p>x-intercept: _____</p> <p>y-intercept: _____</p> <p>End Behaviors: <math>\begin{matrix} \text{As } x \rightarrow \infty, y \rightarrow \\ \text{As } x \rightarrow \infty, y \rightarrow \end{matrix}</math></p>	<p>2-f</p> 

## Lesson Day Warm-Up

- On your mini-whiteboards, write an exponential growth function  $f(x)$  and sketch its graph.
- What are the common features of these graphs? What part of the function creates these common features? What is different about the graphs? What part(s) of the functions cause these differences?
- Now on your mini-whiteboards, sketch the graph of your functions' inverse.
- Then write the function  $f^{-1}(x)$ .
- Now on your mini-whiteboards, write an exponential decay function  $g(x)$  and sketch its graph.
- What are the common features of these graphs? What part of the function creates these common features? What is different about the graphs? What part(s) of the functions cause these differences?
- Now on your mini-whiteboards, sketch the graph of your functions' inverse, then write the function  $g^{-1}(x)$ .

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## Collaborative Activity Instructions:

In your pairs:

### Part 1:

Match each function to its graph, then find the missing characteristics for each function.

Tape/glue each function to its graph (attach them side-to-side)

### Part 2

Match each exponential function/graph to its inverse function/graph. Attach each inverse pair to your poster.

For each pair write a brief explanation of how your group determined that the functions/graphs were inverses of each other.